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The Adaptive Design of Experiments and Markovian Models	5. FUNDING NUMBERS DAAG-55-98-0482
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11. SUPPLEMENTARY NOTES

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13. ABSTRACT (Maximum 200 words)

Research was conducted in three broad areas: setting confidence intervals following an adaptive or sequential experiment; central limit theory for sums of stationary processes; the monotone change problem and related problems in isotonic inference. For the confidence intervals, existing techniques were extended to include grouped sequential methods and to allow for nuisance parameters. The work on central limit theory emphasized state space models (iterated random functions) and non-linear functionals of a linear process. In both cases, asymptotic distributions were obtained under very mild continuity conditions on the function or functional that is summed. The central limit theory was used to obtain the limiting distribution of a new test statistic in the context of a change point problem. For this the change point problem was reformulated to allow several gradual changes, as opposed to the single abrupt change implicit in the classical change point problem. The related problems in isotonic inference include a novel suggestion for the appropriate "degrees of freedom" in an isotonic regression problem.

14. SUBJECT TERMS	15. NUMBER OF PAGES
Central limit theorem, change point problem, confidence intervals, isotonic inference, state space models, very weak expansions.	
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Final Report On
Adaptive Design of Experiments and Markovian Models
DAAG-55-98-0482
Michael Woodroofe

Contents

Statement of the Problems Studied

Research conducted under the grant may be divided into four main categories, each of which is described below.

a) Corrected confidence sets for adaptively designed experiments.

b) Central limit theory for sums of a stationary process.

c) The monotone change problem and related topics from isotonic inference.

d) Miscellaneous other.

Parts *a)* and *b)* were explicitly part of the proposal for DAAG-55–0482, and the monotone change problem in *c)* is an important statistical application of *b)*. The research described in *d)* was not envisioned when the proposal was written.

Summary of Important Results

a) Adaptively Designed Experiments

Corrected confidence intervals for sequentially designed experiments were studied in detail in [4], [5] and [12]. In [5], for example, it was shown that very weak expansions provide good approximations to actual coverage levels for confidence intervals set following a grouped triangular test when responses follow a normal distribution. This was encouraging, because there has been very little previous work on the accuracy of very weak approximations in a grouped sequential setting. The formulations in [4] and [12] are more general, allowing non-normal responses, nuisance parameters, and continuous inspection as well as grouped testing. The problem of setting confidence intervals for an unknown population size is studied in [1]. A main finding is that the problem is much harder in the presence of nuissance parameters, but possible for large populations. In [16] a new version of the non-linear renewal theorem is obtained which in the perturbations are required to

be approximately stationary, but not slowly changing. The limiting joint distribution of the excess over the boundary and the last perturbation is obtained.

b) Central Limit Theory

One central theme of the research was to find limiting distributions for sums of stationary processes and to develop statistical applications. This work took several forms. In [8], asymptotic normality was established for sums $S_n = g(X_1)+\cdots+g(X_n)$ of an iterated random function $X_n = \psi(X_{n-1}, Y_n)$, where $X_0, Y_1, Y_2, \cdots$ are independent and $Y_1, Y_2, \cdots$ are i.i.d.. The conditions in [8] are essentially growth conditions on the L^2 norms of $E(S_n|X_0)$. They do not even require that g be continuous and are applicable the indicators of sets whose boundaries are not too messy. Iterated random functions were explored in more detail in [13]. Central and non-central limit theorems were obtained for sums of non-linear functionals of a linear process. After proper normalization such sums were shown to be either asymptotically normal or to converge in distribution to a multiple Wiener integral, depending on the non-linear function and the sizes of the coefficients in the linear process. This work was reported in [7] and [19]. Applications to density estimation were developed in [18].

c) The Monotone Change Problem
And Related Topics

A new approach to the change point problem was developed in [9]. The new approach allows a gradual change as opposed to the abrupt change implicit in the change point problem; it also allows dependence among the observations. The main contribution is an asymptotic test for stationarity with good power against models with a change that is monotone in an appropriate sense. This work uses the limit theory described above and some related techniques from isotonic inference. The related techniques were developed too. In [2] a novel suggestion for the appropriate "degrees of freedom" following an isotonic regression is made and studied. In [10], [14], and [17] proposals for combining smoothness and monotonicity in the estimation of a non-increasing density are made and studied.

d) Miscellaneous Other

Classical methods for setting confidence intervals often encounter difficulty when parameters are restricted–for example, when a parameter is know to exceed some threshold. Such restrictions do not pose any special difficulty for Bayesian method, but these produce credible sets, not confidence sets. In [11] and [21] Bayesian-frequentist compromises are developed in which Bayesian credible intervals also have high frequentist coverage probability. The specific problems addressed in [11] and [21] arise in high energy physics on one hand and in variance components models on the other. In [15], asymptotic distributions are obtained for the number of clusters in a set of random points.

Publications

Published Research Articles
Peer Reviewed Journals

[1] Choi, Ki Heron, Ruby Wen, and Michael Woodroofe (2001). Sequential confidence intervals for a population size with fixed proportional accuracy. *Sequential Analysis*, **20**, 25-44.

[2] Meyer, Mary and Michael Woodroofe (2000). On the degrees of freedom in shape-restricted regression. *Ann. of Statist.*, **28**, 1083-1104.

[3] Woodroofe, Michael and Hsiuying Wang (2000). The problem of low counts in a signal plus noise model. *Ann. Statist.*, **28**, 1561-1569.

[4] Weng, Ruby and Michael Woodroofe (2000). Integrable expansions for posterior distributions for multi parameter exponential families with applications to sequential confidence levels. *Statistica Silica*, **10**, 693-713.

[5] Weng, Ruby and Michael Woodroofe (2000). Discussion of "Hybrid resampling methods for confidence intervals," by C.S. Chuang and T.L. Lai. *Statistica Sinica*, **10**, 33-37.

[6] Woodroofe, Michael (2001). Discussion of "Sequential analysis: classical problems and new challenges," by T.L. Lai. *Statistica Sinica*, **11**, 380-382.

[7] Wu, Wei Biao (2002). The central limit theorem for functionals of a linear process. To appear in *Statistica Sinica*, **12**.

[8] Wu, Wei Biao and Michael Woodroofe (2000). A central limit theorem for iterated random functions. *Journal of Applied Probability*, **37**, 748-755.

[9] Wu, Wei Biao, Michael Woodroofe, and Graciela Mentz (2000). Isotonic regression: another look at the change point problem. *Biometrika*, **88**, 793-804.

[10] Zhang, Rong, Michael Woodroofe, and Jean Kim (2000). Asymptotic analysis of isotonic estimation for grouped data. *J.S.P.I.*, **98**, 107-118.

[11] Zhang, Tonglin and Michael Woodroofe (2001). Credible and confidence sets for restricted parameter spaces. To appear in *J. Statist. Plan. Inf.*

Other Publications
Theses

[12] Weng, Ruby (1999). *Very Weak Expansions for Sequentially Designed Experiments.* Ph. D. Dissertation, Statistics, The University of Michigan.

[13] Wu, Wei-Biao (2001). *Studies in Times Series and Random Dynamics.* Ph. D. Dissertation, Statistics, The University of Michigan.

[14] Zhang, Rong (2000). *Isotonic Estimation with Smoothing.* Ph. D. Dissertation, Statistics, The University of Michigan.

Submitted Research Articles

[15] Csorgo, S. and Wei-Biao Wu (2002). On the clustering of independent uniform random variables. Submitted to *Random Structures and Algorithms*

[16] Kim, Dong Yun and Michael Woodroofe (2001). Non-linear renewal theory with stationary perturbations. Submitted to *Sequential Analysis.*

[17] Meyer, Mary and Michael Woodroofe (2000). Consistent maximum likelihood estimation using shape restrictions. Submitted to *Statistica Sinica.*

[18] Wu, Wei Biao and Jan Mielniczuk (2000). Kernel density estimation for linear processes. Tentatively accepted by *Ann. Statist.*.

[19] Wu, Wei Biao and Michael Woodroofe (2000). Limit theorems for multiple linear processes. Submitted to *J. Mult. Anal.*

Technical Reports

[20] Kim, Dong Yun (2001). Kim, Dong Yun (2001). Non-linear renewal theory with stationary perturbations. Thesis Proposal.

[21] Zhang, Tonglin (2001). Credible and confidence sets for restricted parameter spaces. Thesis proposal.

Scientific Personnel Supported

The scientific personnel supported by the grant were Dong Yun Kim, Graciela Mentz, Ruby Weng, Michael Woodroofe, Wei-Biao Wu, Rong Zhang, and Tonglin Zhang. Except for Woodroofe, all were Ph. D. students in Statistics. Ruby Weng, Rong Zhang, and Wei-Biao Wu have completed their degrees; their thesis titles are listed above. Tonglin Zhang has completed the research for his thesis and will finish this summer. Dong Yun Kim and Graciela Mentz are expected to finish by the end of the next academic year.